USN | | | | | | | | | | | 06MAT41

Fourth Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Use of statistical tables is permitted.

PART – A

1 a. Find the y(0.1) correct to 6 decimal places by Taylor series method when $dy/dx = xy + 1$, $y(0) = 1.0$. (Consider upto 4^{th} degree term). **(06 Marks)**

b. Using Runge-Kutta method of order 4, compute y(0.2) for the equation, $y' = y - \frac{2x}{y}$, $y(0) = 1.0$ (Take $h = 0.2$). **(07 Marks)**

c. Given that $y' = x^2(1+y)$ and $y(1) = 1.0$, $y(1.1) = 1.233$, $y(1.2) = 1.548$ and $y(1.3) = 1.979$, compute y(1.4) by Adams-Bashforth method. Apply correct formula twice. **(07 Marks)**

2 a. Show that Z^n is analytic. Hence find its derivative. **(06 Marks)**

b. Find a bilinear transformation which maps the points 0, 1, i in the Z-plane onto 1 + i, –i, 2 – i in the W plane. **(07 Marks)**

c. Find the analytic function u + iv, where u is given to be $u = e^x [(x^2 - y^2)\cos y - 2xy \sin y]$. **(07 Marks)**

3 a. Derive Couchy's integral formula in the form
$$f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-a}$$ **(06 Marks)**

b. Expand $f(z) = \frac{7z^2+9z-18}{z^3-9z}$ in the Laurent series that is valid for
i) $|z| > 3$ ii) $0 < |z-3| < 3$. **(07 Marks)**

c. Evaluate $\int_c \tan z\, dz$, where c is $|z| = 2.5$ **(07 Marks)**

4 a. Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

b. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

c. Reduce the differential equation $x\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. Obtain the solution. **(07 Marks)**

PART – B

5 a. Fit a curve of the form $y = ab^x$ for the data given below: **(06 Marks)**

x :	2	4	6	8	10	12
y :	1.8	1.5	1.4	1.1	1.1	0.9

b. Find the coefficient of correlation for the following data: **(07 Marks)**

x :	55	56	58	59	60	60	62
y :	35	38	39	38	44	43	45

c. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body.
  i) What is the probability that mathematics is being studied?
  ii) If a student is selected a random and is found to be studying mathematics, find the probability that the student is a girl. **(07 Marks)**

6 a. Suppose a random variable X takes the values –3, –1, 2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$. Find the value of k and i) find $P[-3 < X < 4]$ and ii) $P[X \le 2]$. **(06 Marks)**

b. Suppose that the student IQ scores form a normal distribution with mean 100 and standard deviation 20. Find the percentage of students whose i) score is less than 80 ii) score falls between 90 and 140, iii) Score more than 120. **(07 Marks)**

c. Obtain mean and variance of binomial distribution function. **(07 Marks)**

7 a. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable 99% confidence limits to the proportion of foggy days in the district? **(06 Marks)**

b. The following table gives the number of bus accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week, using χ^2 test. **(07 Marks)**

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

c. The life X of certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the hypothesis that μ = 800 hours against the alternate hypothesis $\mu \ne 800$ hours at i) 0.5% and 1% level of significance. **(07 Marks)**

8 a. A fair coin is tossed 4 times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. Find the joint distribution function of X and Y. **(06 Marks)**

b. A man's gambling luck follows a pattern. If he wins a game the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance that he wins the first game.
i) Find the transition matrix of the Markov process. ii) Find the probability that he wins the third game. iii) Find out how often, in the long run, he wins. **(07 Marks)**

c. Explain: i) Transient state ii) Absorbing state and iii) Recurrent state by means of an example each. **(07 Marks)**

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06CS42

Fourth Semester B.E. Degree Examination, May/June 2010
Graph Theory and Combinatorics

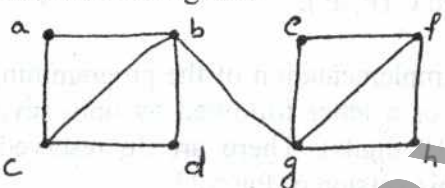
Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Let G = (V, E) be the undirected graph in the Fig.1(a). How many paths are there in G from a to h? How many of these paths have a length 5? **(07 Marks)**

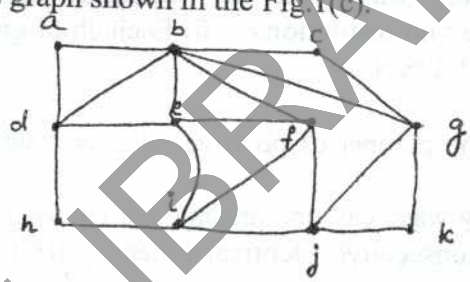
Fig.1(a).



b. Let G = (V, E) be an undirected graph, where $|V| \geq 2$. If every induced subgraph of G is connected, can we identify the graph G? **(06 Marks)**

c. Find an Euler circuit for the graph shown in the Fig.1(c). **(07 Marks)**

Fig.1(c).



2 a. Show that when any edge is removed from K_5, the resulting subgraph is planar. Is this true for the graph $K_{3,3}$? **(07 Marks)**

b. Nineteen students in a nursery school, play a game each day, where, they hold hands to form a circle. For how many days can they do this, with no student holding hands with the same playmate twice? **(07 Marks)**

c. Define chromatic number. What is chromatic polynomial? State the decomposition theorem for chromatic polynomials. **(06 Marks)**

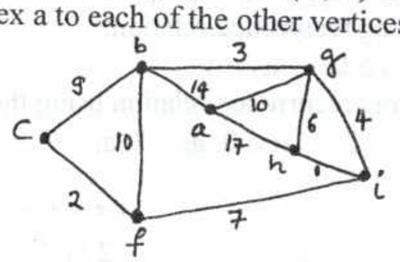
3 a. A classroom contains 25 microcomputers, that must be connected to a wall socket that has four outlets. Connections are made by using extension cords, that have four outlets each. What is the least number cords needed to get these computers set up for class use? **(07 Marks)**

b. Explain the steps in the merge sort algorithm. **(06 Marks)**

c. Using the weights 2, 3, 5, 10, 10, show that the height of the Huffman tree for a given set of weights is not unique. **(07 Marks)**

4 a. Apply Dijkstra algorithm to the weighted graph G = (V, E) shown in Fig.4(a) and determine the shortest distance from vertex a to each of the other vertices in the graph. **(07 Marks)**

Fig.4(a)



b. Use Prim's algorithm to generate an optimal tree for the graph, shown in Fig.4(b). **(06 Marks)**

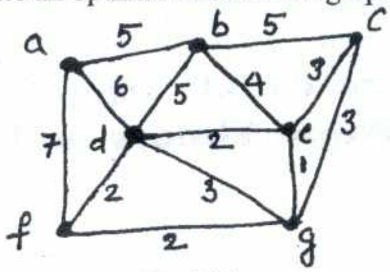


Fig.4(b).

c. Let f be a flow in a network N = (V, E). If C = (P, $\overline{P}$) is any cut in N, then prove that val (f) cannot exceed C (P, $\overline{P}$). **(07 Marks)**

5 a. In a certain implementation of the programming language Pascal, an identifier consists of a single letter or a letter followed by upto seven symbols, which may be letters or digits. (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of Pascal? **(07 Marks)**

b. How many bytes contain i) Exactly two 1's ; ii) Exactly four 1's ; iii) Exactly six 1's and iv) At least six 1's? **(07 Marks)**

c. In how many ways can 10 (identical) dimes be distributed among five children i) If there are no restrictions ; ii) Each child gets at least one dime ; iii) The oldest child gets at least 2 dimes. **(06 Marks)**

6 a. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2, 3, 5. **(07 Marks)**

b. In how many ways can one arrange the letters in CORRESPONDENTS so that i) There is no pair of consecutive identical letters ; ii) There are exactly two pairs of consecutive identical letters. **(07 Marks)**

c. For the positive integers 1, 2, 3, 4, there are n derangements. Define derangements. What is the value of n? **(06 Marks)**

7 a. Give the generating function for :

i) 1, 1, 1, 1,……..1, 1, 1…. all terms are 1

ii) 1, 1, 1, 1,……..1, 0, 0, 0….. first terms are 1, others are 0

iii) 0, 1, 2, 3…….. **(06 Marks)**

b. Find the generating function for $P_d(n)$, the number of partitions of a positive integer n into distinct summands. What is $P_d(6)$ = ? **(07 Marks)**

c. In each of the following, the function f(x) is the exponential generating function for the sequence a_0, a_1, a_2……., whereas the function g(x) is the exponential generating function for the sequence b_0, b_1, b_2……. Express g(x) in terms of f(x) if

i) $b_3 = 3$ and $b_n = a_n$, $n \in N$, $n \ne 3$.

ii) $b_1 = 2$, $b_2 = 4$, and $b_n = 2a_n$, $n \in N$, $n \ne 1, 2$. **(07 Marks)**

8 a. Solve the following recurrence relation :

$a_n = 5a_{n-1} + 6a_{n-2}$ $\quad n \ge 2 \quad a_0 = 1, a_1 = 3.$ **(10 Marks)**

b. Solve the following recurrence relation:

$a_{n+1} - 2a_n = 2^n$, $n \ge 0 \quad a_0 = 1.$ **(10 Marks)**

c. Solve the following recurrence relation using the method of generating functions :

$a_{n+2} - 5a_{n+1} + 6a_n = 2$, $\quad n \ge 0, a_0 = 3, a_1 = 7.$

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USN | | | | | | | | | | | **06CS43**

Fourth Semester B.E. Degree Examination, May/June 2010

Analysis and Design of Algorithms

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Compare the orders of growth of $\log_2(n)$ and $\sqrt{n}$. What is your conclusion? **(06 Marks)**

b. Define O-notation. If $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$, prove that $f_1(n) + f_2(n) \in O(\max\{g_1(n), g_2(n)\})$. **(06 Marks)**

c. Given a positive decimal integer n, write a recursive algorithm which computes the number of binary digits in the binary representation of n. Write the corresponding recurrence relation and solve it. **(08 Marks)**

2 a. Explain the algorithm for selection sort. If A is an array of size n, obtain an expression for the number of key comparisons. **(06 Marks)**

b. Using bubble sort algorithm, arrange the letters of the word 'QUESTION' in alphabetical order. **(06 Marks)**

c. Show how divide and conquer technique can be used to compute the product of two n-digit integers. If n is a power of 2, obtain a recurrence relation for M(n), the number of multiplications and solve it. **(08 Marks)**

3 a. What are the three major variations of decrease and conquer technique? Explain each with an example. **(06 Marks)**

b. Sort the letters of the word "EXAMPLE" in alphabetical order using insertion sort. **(06 Marks)**

c. Describe the Johnson Trotter algorithm for generating permutations. Generate all permutations of {3, 5, 7} using the following:

i) Bottom up minimal change algorithm

ii) Johnson Trotter algorithm. **(08 Marks)**

4 a. Write an algorithm for DFS. With an example, explain how this algorithm can be used to solve topological sorting problem. **(10 Marks)**

b. Using quick sort, arrange the letters of the word "QUICKSORT" in alphabetical order. Show all the steps clearly and draw the tree of the recursive calls made. **(10 Marks)**

PART – B

5 a. Define a 2-3 tree. For any 2-3 tree of height h consisting of n nodes, prove the following:

$$\log_3(n+1) - 1 \le h \le \log_2(n+1) - 1$$ **(06 Marks)**

b. Describe the algorithm for heap sort. **(06 Marks)**

c. Show how Horspool's algorithm can be used to search for the pattern BARBER in a given text. Consider all the four cases. **(08 Marks)**

6 a. Apply Warshall's algorithm to find the transitive closure of the graph defined by the following adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(05 Marks)

b. Using Floyd's algorithm, solve the all-pairs shortest path problem for the graph whose weight matrix is given below:

$$\begin{pmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{pmatrix}$$

(10 Marks)

c. Using Kruskal's algorithm, obtain a minimum cost spanning tree for the graph Fig.Q6(c) given below: **(05 Marks)**

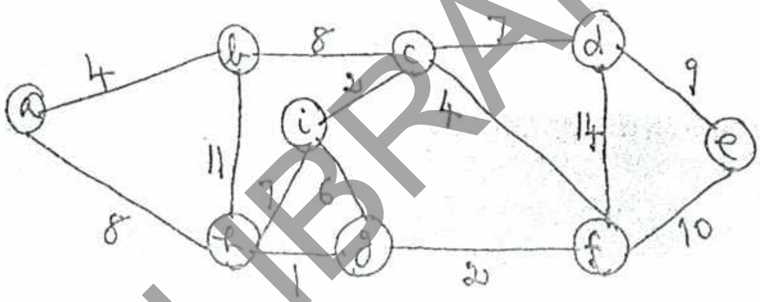

Fig.Q6(c)

7 a. Construct a Huffman code for the following data: **(06 Marks)**

Character	A	B	C	D	E
Probability	0.4	0.1	0.2	0.15	0.15

Decode the text whose encoding is 100010111001010 using the above Huffman code.

b. Write short notes on P, NP and NP-complete problems. **(06 Marks)**

c. Explain how backtracking is used for solving 4 – queens problem. Show the state space tree. **(08 Marks)**

8 a. Solve the following instance of knapsack problem using branch and bound algorithm:

Item	1	2	3	4
Weight	4	7	5	3
Value	\$ 40	\$ 42	\$ 25	\$ 12

The capacity of the knapsack is W = 10. **(08 Marks)**

b. When does collision occur in hashing? What are the different mechanisms used to resolve collisions? **(04 Marks)**

c. What are decision trees? Explain how decision trees are used in sorting algorithms. **(08 Marks)**

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06CS44

Fourth Semester B.E. Degree Examination, May/June 2010

Object Oriented Programming with C++

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain the various features of object oriented programming. **(10 Marks)**

b. Discuss function prototyping, with an example. Also write its advantages. **(05 Marks)**

c. Define the 'this' pointer, with an example, indicate the steps involved in referring to members of the invoking object. **(05 Marks)**

2 a. What are friend non-member functions and friend member functions? Explain with suitable examples. **(08 Marks)**

b. Write a C++ program to count the number of objects of a certain class. **(06 Marks)**

c. Write a note on namespaces. **(06 Marks)**

3 a. What is dynamic memory management? Write a C++ program demonstrating the usage of new and delete operators for a single variable as well as for an array. **(10 Marks)**

b. What are constructors and destructors? Explain the different types of constructors in C++, with examples. **(10 Marks)**

4 a. Discuss with examples, the implications of driving a class from an existing class by the 'public' and 'protected' access specifiers. **(08 Marks)**

b. What is function overriding? Give an example. Justify the statement: "function overriding is a form of function overloading". **(06 Marks)**

c. Write a C++ program to initialize base class members through a derived class constructor. **(06 Marks)**

PART – B

5 a. Define and give the syntax for the following :

i) Virtual function ; ii) Pure virtual function ; iii) Abstract base class. **(06 Marks)**

b. What is a virtual table? How does it help in implementing dynamic polymorphism? Explain with an example. **(08 Marks)**

c. Draw the class hierarchy for handling streams in C++. How is text input achieved in C++? **(06 Marks)**

6 a. What is a stream? What are the various flags and functions associated with error handling of streams in C++? **(08 Marks)**

b. What is operator overloading? Explain with examples the circumstances under which operator overloading becomes mandatory. **(12 Marks)**

7 a. Create a class called 'distance' with data member's feet and inches and appropriate constructor (s). Overload the greater than operator (>) for the distance class to tackle the following conditions : i) $d_1 > d_2$; ii) $d_1 >$ float ; iii) Float $> d_1$, where d_1 and d_2 are objects of the distance class and float is a floating point value representing distance (ex : 4.5 means 4 feet 6 inches). **(08 Marks)**

b. Create a class called 'string' with a data member to hold a string and a constructor to set it. Overload the subscript to set it. Overload the subscript operator for the string class to accept a character as a parameter and return the position of its first occurrence, if found, else a negative value. **(06 Marks)**

c. Explain with examples, the conversion from basic type to class type and class type to basic type. **(06 Marks)**

8 a. Define a function template giving its syntax. Write a C++ program to implement array representation of a stack for integers, characters and floating point numbers using class template. **(12 Marks)**

b. Explain the C++ style solution for handling exceptions. **(08 Marks)**

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
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Fourth Semester B.E. Degree Examination, May/June 2010
Microprocessors

Time: 3 hrs. Max. Marks:100

Note: ***1.Answer any FIVE full questions, selecting at least TWO questions from each part***
2.ALP should be well commented.

PART – A

1 a. Explain the internal architecture of 8086, with a neat diagram. **(10 Marks)**
b. What is meant by pipelining? How is it implemented in 8086? Explain the advantages of pipelining. **(05 Marks)**
c. Illustrate the concept of segmented memory, with a neat diagram. Explain four advantages of segmentation. **(05 Marks)**

2 a. List any six assembly language program development tools. Explain any four ALP development tools. **(10 Marks)**
b. Construct the machine code for MOV CL, [BX] instruction. **(10 Marks)**

3 a. Briefly explain various addressing modes of 8086, with suitable instructions. **(08 Marks)**
b. Explain with an example, how multiple If-Then-Else statement can be implemented, using ALP. **(08 Marks)**
c. Write an ALP to clear all control flags of 8086. **(04 Marks)**

4 a. Differentiate between a macro and subroutine. **(04 Marks)**
b. Explain with an example, how parameters can be passed to a subroutine, using stack. **(08 Marks)**
c. Write an ALP to validate a password. Assume the correct password as SECRET. **(08 Marks)**

PART – B

5 a. Explain with examples, the following assembler directives:
i) EXTRN ii) EVEN iii) TYPE iv) ASSUME. **(10 Marks)**
b. Compute the factorial of a given 8-bit number using recursion. **(10 Marks)**

6 a. Illustrate with a neat diagram, the working of 8086 in the minimum mode. Also give the timing diagram of I/O read operation. **(10 Marks)**
b. Interface four 8 KB RAMS starting with an address of 60000H. Draw the memory map and address decoder worksheet. Use 74LS138 decoder for external address decoding. **(10 Marks)**

7 a. List and describe the steps a 8086 will take when it responds to an interrupt. **(06 Marks)**
b. Briefly explain the operation of 8259, with a neat block diagram. **(08 Marks)**
c. Describe the response a 8086 will make , if it receives an NMI interrupt signal during a division operation which produces a divide by zero interrupt. Illustrate this concept with a stack diagram. **(06 Marks)**

8 a. Draw the control word format of 8255. Explain it. **(08 Marks)**
b. Explain different methods of data transfer schemes, with suitable examples. **(06 Marks)**
c. Write an ALP to display 0 to 9 on a 7-segment LED display device. **(06 Marks)**

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06CS46

Fourth Semester B.E. Degree Examination, May/June 2010
Computer Organization

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. PC contains the address of the instruction stored in main memory of the computer. The instruction is "MOVE (R3), R2". List the steps needed to execute the machine instruction MOVE (R3), R2. **(08 Marks)**

b. Explain with examples, all the generic addressing modes, with assembler syntax. **(12 Marks)**

2 a. Convert the following pairs of signed decimal numbers to 5 bit 2's. Complement the numbers and add them. State whether overflow occurs or not.
i) -14 and 11 ii) -10 and -13 iii) -3 and -8. **(06 Marks)**

b. What is word alignment of a machine (microprocessor based system)? Explain. What are the consecutive addresses of aligned words for 16, 32 and 64 bit word lengths of machines? Give two consecutive addresses for each case. **(05 Marks)**

c. Bring out the five key differences between subroutine and interrupt service routine. **(05 Marks)**

d. What is the function of an assembler directive? Give two examples of assembler directives used for the reservation of memory locations for variables. State their functions. **(04 Marks)**

3 a. Define and explain briefly the following:
i) interrupt.
ii) vectored interrupt.
iii) interrupt nesting.
iv) an exception and give two examples. **(13 Marks)**

b. Explain in brief, with the help of a diagram, the working of daisy chain with multiple priority levels and multiple devices in each level. **(07 Marks)**

4 a. In a computer system, PCI bus is used to connect devices to the processor (system bus) bus. Consider a bus transaction in which the processor reads four 32-bit words from the memory. Explain the read operation on the PCI bus between memory and processor. Give signal and timing diagram. **(12 Marks)**

b. Draw the block diagram of universal bus (USB) structure connected to the host computer. Briefly explain all fields of packets that are used for communication between a host and a device connected to an USB port. **(08 Marks)**

PART – B

5 a. Define and explain the following :
 i) Memory access time ii) Memory cycle time
 iii) Random access memory (RAM) iv) Static memories. **(04 Marks)**

b. Differentiate the static RAM (SRAM) and dynamic RAM (DRAM) giving four key differences. State the primary usage of SRAM and DRAM in contemporary computer systems. **(04 Marks)**

c. Define memory latency and bandwidth in case of burst operation that is used for transferring a block of data to or from synchronous DRAM memory unit. **(05 Marks)**

d. Draw a neat block diagram of memory hierarchy in a contemporary computer system. Also indicate relative variation of size, speed and cost per bit, in the hierarchy. **(07 Marks)**

6 a. Explain a simple method of translating virtual address of a program into physical address, with the help of a diagram. **(08 Marks)**

b. Explain structural organization of moving head magnetic hard disk, with multiple surfaces for storage of data. Explain how moving head assembly works for reading data. **(06 Marks)**

c. Answer the following with respect to the magnetic disk, the secondary storage device:
 i) seek time
 ii) latency
 iii) access time **(06 Marks)**

7 a. In carry – look ahead addition, explain generate G_i and propagate P_i functions for stage i with the help of Boolean expression for G_i and P_i . **(04 Marks)**

b. Perform signed multiplication of numbers -12 and -11 using both multiplication algorithm. Represent the numbers in 5-bits including sign bit. Give booth multiplier recoding table that is used in the above multiplication. **(08 Marks)**

c. Perform division of number 8 by 3 (8 ÷ 3) using non-restoring division algorithm. **(08 Marks)**

8 a. Draw the block diagram of the three-bus organization of data path, which providess multiple internal paths to enable several transfers to take place in parallel. Label the registers and functional components of the processor and their connection to the respective bus of data path. **(16 Marks)**

b. Draw a block diagram of a complete processor and identify the units. **(04 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, May/June 2010
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the projection of the line AB on CD where $A = (1, 2, 3)$, $B = (-1, 0, 2)$, $C = (1, 4, 2)$, $D = (2, 0, -1)$. **(06 Marks)**

b. Find the angle between two lines whose direction cosines are given by $l + 3m + 5n = 0$ and $2mn - 6nl - 5lm = 0$. **(07 Marks)**

c. A line makes angles $\alpha, \beta, \gamma, \delta$ with diagonals of a cube. Prove that $\text{Cos}^2\alpha + \text{Cos}^2\beta + \text{Cos}^2\gamma + \text{Cos}^2\delta = \frac{4}{3}$. **(07 Marks)**

2 a. Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to $5x + y + 4z = 0$. **(06 Marks)**

b. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, -1, -1) are coplanar. Find the equation of the plane containing them. **(07 Marks)**

c. Find the equation of a straight line through (7, 2, -3) and perpendicular to each of the lines. $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 3(\sqrt{3}\,\hat{i} - \hat{j})$, $\vec{b} = 6\hat{j}$, $\vec{c} = 3(\sqrt{3}\,\hat{i} + \hat{j})$ form an isosceles triangle. **(06 Marks)**

b. A particle moves along the curve $\vec{r} = 3t^2\hat{i} + (t^3 - 4t)\hat{j} + (3t + 4)\hat{k}$. Find the components of velocity and acceleration at $t = 2$ in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

c. Find the angle between the normals to the surfaces $x^2y^2 = z^4$ at (1, 1, 1) and (3, 3, -3). **(07 Marks)**

4 a. Find the directional derivatives of the function $\phi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point (1, 1, 1). **(06 Marks)**

b. Find the div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. If $\vec{v} = 2xy\,\hat{i} + 3x^2y\,\hat{j} - 3ayz\,\hat{k}$ is solenoidal at (1, 1, 1), find a. **(07 Marks)**

5 a. Find the unit normal vector to the surface $xy + x + zx = 3$ at (1, 1, 1). **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Find the constants 'a', 'b', 'c' such that the vector field
$(\text{Sin } y + az)\ \hat{i} + (bx \text{ Cos } y + z)\ \hat{j} + (x + cy)\ \hat{k}$
is irrotational. Also find the scalar field ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\nabla^2 (\log r) = \frac{1}{r^2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. **(07 Marks)**

6 a. Find the Laplace transform of Sin 2t Sin 3t. **(05 Marks)**

b. Find $L\left[\frac{(1-e^t)}{t}\right]$. **(05 Marks)**

c. Find $L\left[e^{-t}(3 \text{ Sinh } 2t - 2 \text{ Cosh } 3t)\right]$. **(05 Marks)**

d. Find the Laplace transform of $f(t) = \begin{cases} t/\lambda & \text{when} \quad 0 < t < \lambda \\ 1 & \text{when} \quad t > \lambda \end{cases}$. **(05 Marks)**

7 a. Evaluate $\int_0^\infty \frac{\text{Sin} t}{t} dt$ using Laplace transform. **(05 Marks)**

b. Find the inverse Laplace transform of $\frac{1}{(s^2+3s+2)(s+3)}$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s-1}{s^2-6s+25}\right]$. **(05 Marks)**

d. Find $L^{-1}\left[\log\left\{\frac{s^2+1}{s^2-s}\right\}\right]$. **(05 Marks)**

8 a. Find $L^{-1}\left[\frac{1}{s^2(s+5)}\right]$ using convolution theorem. **(10 Marks)**

b. Solve the differential equation $y'' + 2y' + y = 6te^{-t}$ under the condition $y(0) = 0 = y'(0)$ using Laplace transform. **(10 Marks)**

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